# Impact of the NaCl Solute on the Occurrence of the Mpemba Effect

## *How do different concentrations of the NaCl solute affect the occurrence of the Mpemba Effect?*

**Subject:** Physics

2911 words

**May 2021**

# Contents

# Chapter 1: Introduction

## 1.1 Background Information

Mpemba effect was first discovered in 1963 by a Tanzanian schoolboy Ernesto Mpemba while he was making ice cream. In a rush to get the space in freezer for his ice cream mixture, Mpemba decided not to let it cool down before placing it into the freezer. Surprisingly, his mixture froze before any other mixture despite having been placed shortly after it was boiled. Consequently, the effect was then described in a paper by professor Osborne (Mpemba & Osborne, 1969), according to which it was initially defined and later on summarized by Jeng (2006, p. 514) as follows:

> “If the two bodies of water, identical in every way, except that one is at a **higher temperature** than the other are exposed to identical **subzero conditions**, the initially hotter water will freeze first.”

In recent years, Mpemba effect has seen a great increase in the number of plausible theories trying to offer an explanation behind its occurrence and in turn provide a deeper understanding of underlying physics at play. Even so, the effect still remains quite simple to describe but hard to predict and explain. In many respects, it is this mysterious and contradicting aspect of Mpemba effect that has intrigued dozens of scientists as well as inspired me to explore it further.

Among the numerous theories trying to offer potential explanations behind the Mpemba effect, *Theory of Solutes* claims that the impact of solutes is what causes the initially hotter water to freeze faster than the cooler water. Katz (2009) suggested that the origin of the Mpemba effect was due to freezing-point depression by solutes, either gaseous or solid, whose solubility decreases with increasing temperature so that they are removed when water is heated.

In an attempt to explore the theory as well as the practical side, this work will turn to examining the original paper that first described the effect (Mpemba & Osborne, 1969), study done by Katz (2009) and Thomas (2007). In addition to this, work of Panković and Kapor (2012) will be considered.

## 1.2 Choice of Topic

As the Mpemba effect implies that initially hotter water freezes faster than colder water, it naturally follows that identifying the circumstances under which this takes place might give rise to some important implications for the practice. Concretely, Mpemba effect might be used to speed up certain processes, ranging from simple instances of making ice-cream where Mpemba first observed the effect to more advanced cases. Additionally, furthering the research and challenging the reproducibility of this topic might lead to a better understanding of the Mpemba effect. Finally, more research might prove useful for settling the existing disagreements in the scientific communities surrounding the origin of the effect itself.

## 1.3 Research Question

In order to further explore the Mpemba Effect this essay will look into the *Theory of Solutes* and consider its applicability in explaining the phenomenon using a specific case. More precisely, this work will consider (a) **whether the initial temperatures influence the speed of cooling** and (b) **how different concentrations of the NaCl solute affect the occurrence of the Mpemba Effect?**

## 1.4 Literature Review

Counterinuitive, as it appears to violate the First Law of Thermodynamics, this effect has seen a resurgence in the last few years despite having been around for quite some time, even dating back to Aristotle (Lee H. D. & Aristotle, 1962). In turn, this led to the development of different theories, none of which are unequivocally accepted. On the other hand, there is some recent research which completely renounces the Mpemba effect, claiming that it is not a genuine physical effect and thus constitutes a scientific fallacy (Burridge & Linden, 2016; Burridge & Hallstadius, 2020).

In trying to explain the effect, as it has been observed dozens of times, multiple approaches were considered by scientists; namely, (1) supercooling, (2) convection currents and (3) theory of solutes.

However, as put by Jeng (2006), in order to examine the effect "it is important to consider a parameter that might change during the experiment." Furthermore, this might give insight into why the hotter sample would not have the same properties when it reaches the initial temperature of the cooler sample.

### 1.4.1 Newton's Law of Cooling

Newton's Law of Cooling is important to the study of water freezing as it allows to theoretically predict its behaviour. This can be modelled using the following equation:

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = -k \cdot (T - T_{amb}) \tag{1.1}$$

where $T$ is temperature of a sample, $k$ is an experimental constant, and $T_{amb}$ is ambient temperature of a freezer such that $T \geq T_{amb}$. From this, it may noted that the bigger the temperature difference between $T$ and $T_{amb}$ the faster the water sample freezes. However, as explained by Thomas (2007, p. 2) this does not support the effect; it rather just reinstates that the hotter water must traverse the same temperature path as the cooler water.

In addition, Panković and Kapor (2012) proposed a theoretical model based on the Newton's Law of Cooling and implied that it can predict when the Mpemba effect will occur. This model, however, is not widely accepted since the Mpemba effect is theorized to occur as a result of different factors - for which the Newton's Law of Cooling cannot really account for.

### 1.4.2 Freezing Point Depression

Increasing the concentration of table salt, and thus its main component NaCl, decreases the freezing point of water, leading to a phenomenon called the freezing point depression (Helmenstine, n.d.). On the quantitative side this is described by the Clausius-Clapeyron equation and Raoult's Law whereby:

$$F_{tot} = F_{solv} - \Delta T_f, \tag{1.2}$$

where $F_{tot}$ is the freezing point of the total mixture, $F_{solv}$ is the freezing point of solvent (water) and $\Delta T_f$ is the change in temperature. From this it follows that the freezing point of water is affected by $\Delta T_f$ and decreases as a consequence of $\Delta T_f$ increasing. $\Delta T_f$ itself is defined as:

$$\Delta T_f = \text{molality} \cdot K_f \cdot i \tag{1.3}$$

where molality refers to the measure of number of moles of solute present in 1 kg of solvent, $K_f$ is the cryoscopic constant and $i$ is the Van't Hoff factor. In case of tap water, cryscopic constant, $K_f$, is $1.853\,\mathrm{K\,kg\,mol^{-1}}$ and the Van't Hoff factor, $i$, for NaCl is found to be 2 ("Van't Hoff Factor", n.d.) which implies that the change in freezing point is influenced by the molality

# Chapter 2: Experimental Design

## 2.1 Variables

### 2.1.1 Independent Variable

Mass concentration, $\rho_i$, is a **an amount of substance** within a given **mixture**, respectively referred to as a solute and a solvent. Mathematically, it follows:

$$\rho_i = \frac{m_{solute}}{V} \tag{2.1}$$

where $m_{solute}$ is mass of the solute (salt) and $V$ is the volume of the solvent (water). Mass concentration, $\rho_i$, is the independent variable and is changed by changing the mass of the table salt, $m_{solute}$, while the volume $V$ remains constant. It follows that by increasing the value of $m_{solute}$ in the range of $1.0\,\mathrm{g} - 5.0\,\mathrm{g}$, the value of $\rho_i$ increases as well.

### 2.1.2 Dependent Variable

In as such, the cooling rate of the sample, $\frac{\mathrm{d}T}{\mathrm{d}t}$, represents a dependent variable as it is affected by other factors, namely salt concentration of a given sample as well as the initial temperature, $T_i$. It follows from the hypothesis that by increasing the concentration of salt, the value of $\frac{\mathrm{d}T}{\mathrm{d}t}$ will increase as well.

### 2.1.3 Controlled Variables

| **Controlled Variables** | **Description** | **Value** |
|---|---|---|
| *Ambient Temperature* ($T_{amb}$) | Affects the cooling time of the samples inside freezer. It was controlled by allowing the temperature to drop to $-20\,^{\circ}\mathrm{C}$ after every finished trial. | $(-20 \pm 0.5)^{\circ}\mathrm{C}$ |
| *Initial Temperature of Water* ($T_i$) | Initial temperatures of $T_i = 28\,^{\circ}\mathrm{C}$ and $T_i = 50\,^{\circ}\mathrm{C}$ were used as starting values for the measurements. Sample was allowed to cool down until the given temperature and the data was recorded | $T_i = 28\,^{\circ}\mathrm{C}$ and $T_i = 50\,^{\circ}\mathrm{C}$ |
| *Volume of the Water* ($V$) | Volume of water directly affects the time taken to cool down a sample. All samples were 40 mL and measured using a beaker prior and after boiling. | $(40 \pm 0.5)\mathrm{mL}$ |
| *Placement in the Freezer* | The position of the sample changes the cooling efficiency of the freezer. As such, all samples were placed *approximately* in the center of the freezer. | - |
| *Placement of Thermistor inside Water* | Thermistor was fully submerged *approximately* in the middle of the the sample, 1 cm from the surface. | - |
| *Freezer Floor Insulation* | Insulation affects the rate of transfer of heat. Paper towels were placed between the cups and the freezer floor to minimize the frost and thermally insulate the sample . | - |

**Table 1:** Controlled Variables

## 2.2 Hypothesis

Firstly, if the Mpemba effect is to be observed then the initially hotter sample ($T = 50\,°\mathrm{C}$) has to reach 0 °C before the initially cooler sample ($T = 28\,°\mathrm{C}$). Secondly, if the salt concentration in tap water samples of 40 mL increases from 1.0 g to 5.0 g, then the time taken for the samples to reach 0 °C will decrease, which might lead to the Mpemba effect being observed.

## 2.3 Equipment

In this study the following equipment was used:

- Small freezer with an internal temperature of $-20.1\,°\mathrm{C}$ to $-18.2\,°\mathrm{C}$
- Small plastic cups (PP05) measuring: top radius ($8.92 \pm 0.01$cm), bottom radius ($6.21 \pm 0.01$cm) and height ($9.76 \pm 0.01$cm)
- Vernier caliper (±0.01 cm)
- Glass Beaker (±0.5 mL)
- Tap water
- Table salt
- Scale (±0.01 g)
- Arduino Uno Rev3 (*Arduino Uno Rev3*, 2019)
- Insulated temperature probe (NTC Thermistor)
- Resistor with nominal resistance 1 kΩ
- Paper towels
- Digital data logger (Laptop)

## 2.4 Experimental Methods

In this experiment, 40 mL samples of regular tap water were used with additionally added table salt (1.0 g - 5.0 g) with increments of 1.0 g. Water samples without additional salt were used as a control. To obtain the experimental data, an Arduino was set up and programmed to calculate the temperature of a thermistor.

### 2.4.1 Setup

Arduino was set up to collect relevant data and determine the potential difference of a thermistor. This was achieved by creating a voltage-divider over a regular 1 kΩ resistor (experimentally 989 Ω) and a 10 kΩ thermistor depicted on Fig. 1. Thermistor's specifications were experimentally found as shown in Table 2. The circuit was completed and 5 V potential difference established over the circuit by using Arduino's 5 V and GND (Ground) ports as shown below.

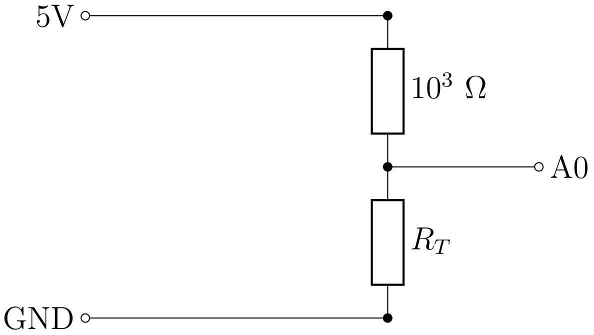

**Figure 1:** Circuit of resistor and thermistor $R_T$ connected to Arduino's analog port (A0)

| **Temperature** (°C)<br>(±0.1 °C) | **Resistance** (Ω)<br>(±1 Ω) |
|---|---|
| 22.6 | 1330 |
| 11.3 | 2140 |
| 9.5 | 2320 |
| −13.7 | 7070 |

**Table 2:** Values of $R_T$ at specific temperatures

From Fig. 1, the circuit has a resistance $R_{tot} = R + R_T$, where $R$ is resistor with nominal resistance 1 kΩ and $R_T$ is the thermistor. From this it follows that the voltage of the thermistor, $V_T$, can be easily by using the Ohm's Law as (see Appendix for derivation):

$$V_T = V_{arduino} \cdot \frac{R_T}{R + R_T}. \tag{2.2}$$

In order to establish a relationship between resistance and temperature, calibration of the thermistor was done by using the Steinhart-Hart $\beta$ equation (Steinhart & Hart, 1968). By doing so, the voltage data was could processed in real-time using the Python code (refer to the Appendix) thereby returning the temperature of the sample.

### 2.4.2 Procedure

Initially, the 40 mL samples of tap water were boiled and salt was added. After mixing the samples with salt, they were left rest for a few minutes and then placed into the freezer.

All samples were positioned *approximately* in the center of the freezer and thermally insulated by paper towels to prevent frosting with freezer floor. This was done to account for the freezer's efficiency as well as of the subsequent trials

The ambient temperature of the freezer was allowed to drop back to −20 °C after every measurement according to Ibekwe & Cullerne (2016). Briefly, the experiment was conducted in the following steps:

1. Using a scale, 1.0 g of table salt was measured and placed into a plastic cup. For subsequent measurements, table salt was measured in 1.0 g increments in the range 1.0 g - 5.0 g
2. Tap water was heated up until the boiling point by using a kettle. (Kettle was previously thoroughly cleaned so as not to contain any source of impurities)

3. Boiled tap water was then sampled (40 mL) using a glass beaker and transferred into the same plastic cup

4. Plastic cup was hand-stirred until the table salt completely dissolved and left to rest for a few minutes at room temperature

5. When the sample reached 50 °C, thermistor was placed in the middle of the sample, approximately 1 cm from the surface level.

6. Sample with the thermistor was then placed approximately in the center of the freezer and the data collecting process on the laptop started. (Freezer floor was insulated by using paper towels)

7. Sample was left in the freezer until it reached −5 °C (prompted by Python program), when the measuring stopped and sample was taken out.

8. After the procedure had been repeated for all mass concentrations, almost identical procedure was done with the 28 °C (These samples were let to cool down to 28 °C after which they were placed in the freezer with the thermistor)

### 2.4.3 Safety Considerations

During the experiment trials, special care was taken while handling hot water samples in order to avoid hand burn. As a precautionary measure, heat-resisting gloves were used. Similarly, when handling cooled samples the same gloves were used. No other biological or environmental hazards were identified.

# Chapter 3: Results & Discussion

As from Table 3 and Table 4, Mpemba effect was not observed for the control samples (0 mg mL$^{-1}$). The time taken for the cooler sample to reach 0 °C was 2030 s whereas the hotter sample took 2980 s. **This shows that the initially cooler water did indeed reach** 0 °C **before the initially hotter water.**

Results further show that for both initial temperatures $T_i$ (28 °C and 50 °C), increasing the mass concentration of salt, $\rho_i$, from 0 to 125 mg L$^{-1}$, resulted in a decreases of the elapsed time until the samples reached 0 °C. In particular, the 2030 s to 1940 s (for $T_i = 28\,^\circ\mathrm{C}$), as shown in Table 3, and from 2980 s to 1950 s (for $T_i = 50\,^\circ\mathrm{C}$) noted in Table 4.

Accounting for the anomalies shown in bold, it should be noted that those samples appear to be out of the proposed trend. In those cases, it might be proposed that the samples supercooled and were thus able to cool faster. However, in both cases in Table 3 and Table 4, those behaviors occur without a particular pattern which further reinstates that it might be an oddity that is best understood through the supercooling behavior of water.

| **Initial Temperature** $T_i$ (°C) | **Mass of Salt** $m$ ($10^3$ mg) | **Volume of Sample** $V$ (mL) | **Mass Concentration** $\rho_i$ (mg mL$^{-1}$) | **Elapsed Time until** 0 °C $t$ (s) |
|---|---|---|---|---|
| 28 | 0 | 40 | 0 | 2030 |
| | 1 | | $25 \pm 0.563$ | 2020 |
| | 2 | | $50 \pm 0.875$ | 2000 |
| | 3 | | $75 \pm 1.19$ | **1940** |
| | 4 | | $100 \pm 1.50$ | 1990 |
| | 5 | | $125 \pm 1.81$ | 1940 |

**Table 3:** Data collected for initial temperature $T_i = 28\,^\circ\mathrm{C}$. Anomalies are shown in bold

| **Initial Temperature** $T_i$ (°C) | **Mass of Salt** $m$ ($10^3$ mg) | **Volume of Sample** $V$ (mL) | **Mass Concentration** $\rho_i$ ($\mathrm{mg\,mL^{-1}}$) | **Elapsed Time until** 0 °C $t$ (s) |
|---|---|---|---|---|
| 50 | 0 | 40 | 0 | 2980 |
| | 1 | | $25 \pm 0.563$ | 2670 |
| | 2 | | $50 \pm 0.875$ | **2100** |
| | 3 | | $75 \pm 1.19$ | 2360 |
| | 4 | | $100 \pm 1.50$ | 2170 |
| | 5 | | $125 \pm 1.81$ | 1950 |

**Table 4:** Data collected for initial temperature $T_i = 50\,^\circ\mathrm{C}$. Anomalies are shown in bold.

Further comparing the elapsed times until 0 °C between Table 3 and 4, we may note that the Mpemba effect was not observed for any mass concentration, as all initially cooler samples cooled down to 0 °C first. However, it should be noted that for the concentration of $125\,\mathrm{mg\,mL^{-1}}$, the effect was very close to being observed. In fact, while the cooler sample reached 0 °C in 1940 s, the initially hotter sample reached it in 1950 s, with a mere 10 s difference.

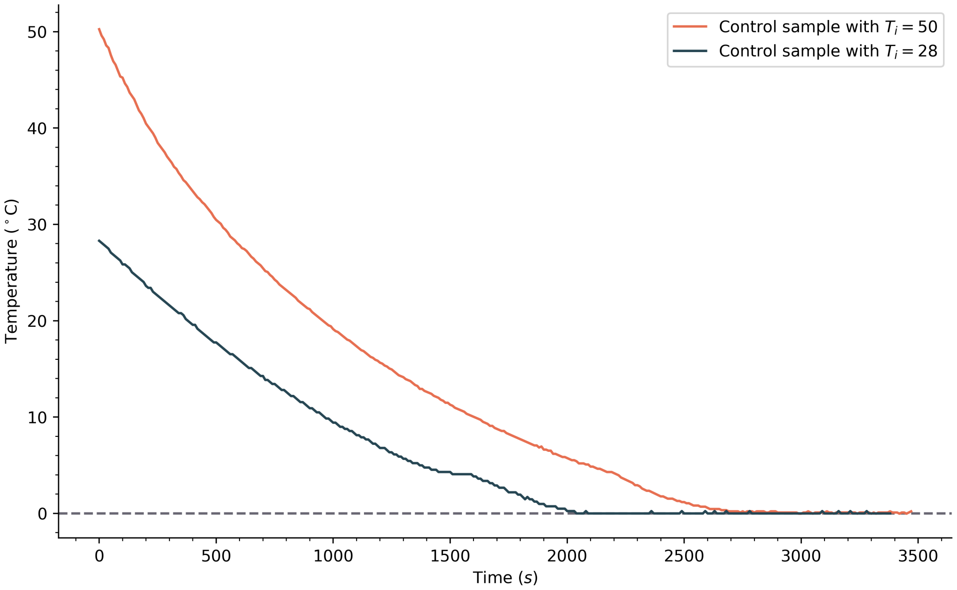

**Figure 2:** Control samples at $T_i = 50\,^\circ\mathrm{C}$ (orange) and $T_i = 28\,^\circ\mathrm{C}$ (blue).

Although 125 mg mL$^{-1}$ did not observe the Mpemba effect, their behavior below 0 °C was very different from the rest. As seen in Fig. 3, initially hotter sample reached the colder temperatures before the initially cooler sample. This behavior was also observed by Ibekwe and Cullerne (2016) for similar group samples, whereby they claimed to have observed the Mpemba effect. It is worth noting that both Ibekwe and Cullerne (2016) and Jeng (2006) chose the point where the ice forms as the freezing point. This study considered the point 0 °C as the freezing point for all samples, for the sake of repeatability and in order to be more in line with the original research done by Mpemba and Osborne (1969).

In comparison to the control samples (Fig. 2), samples with 125 mg mL$^{-1}$ (Fig. 3) exhibited a significantly different behavior. Whereas the two 125 mg mL$^{-1}$ samples reached the 2000 s mark at approximately the same time; temperature curves of the control samples continued running parallel during the duration of the experiment, with cooler sample reaching 0 °C first.

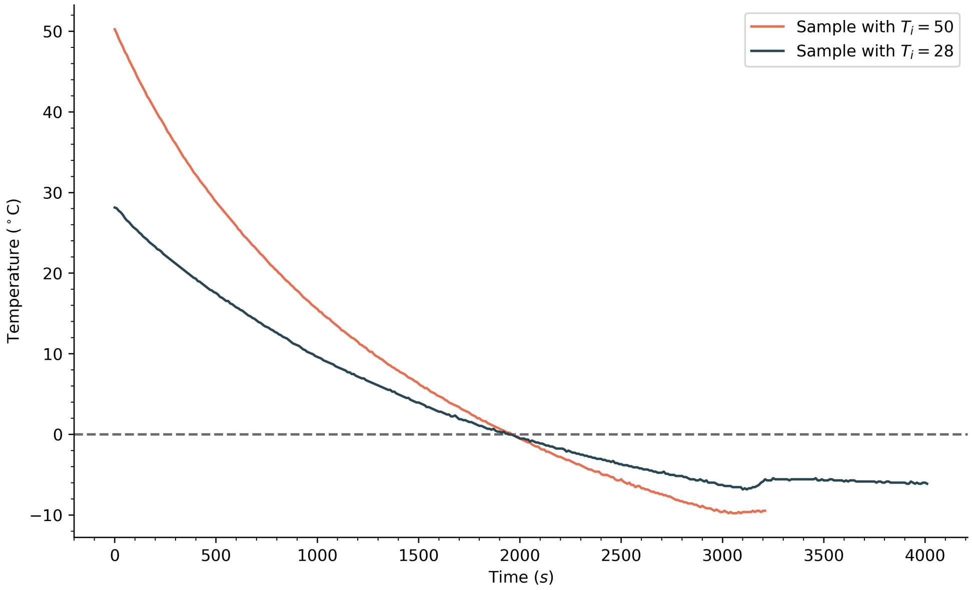

**Figure 3:** 5 g samples at $T_i = 50\,°\mathrm{C}$ (orange) and $T_i = 28\,°\mathrm{C}$ (blue).

Overall, the salted samples were observed to have lower freezing temperatures as compared to the control samples. Furthermore, this implied that the rate of change of temperature for salted samples was greater and thus resulted in steeper temperature curve. Conversely, gradients for the control samples were observed to be less steep for the same reasons.

Furthermore, it was noted that the time taken for the hotter sample took only 0.515% more time to reach 0 °C in comparison to the cooler sample. Ibekwe and Cullerne (2016) observed a difference of 14% between hotter sample freezing first and cooler sample freezing afterwards.

In addition, as noted by Jeng (2006), "Finding that the Mpemba effect does not occur under certain conditions is a good experimental result." In another study by Ibekwe and Cullerne (2016) it was claimed that the convection currents circulate the warmest water to the surface, accelerating these mechanisms and that above approximately 45 °C they are sufficiently accelerated to enable a body of water to overtake a cooler body of water, reaching 0 °C and freezing first. In comparison with the theory of solutes, the convection currents might offer a valid explanation to the observed processes.

# Chapter 4: Conclusion

## 4.1 Conclusion

In this essay, I sought to explore and offer an answer to the following research question: (a) does hot water freeze faster than cooler water and (b) how do different concentrations of NaCl solute affect the occurrence of the Mpemba Effect. Results of the investigation imply that the cooler water reached 0 °C before the initially hotter water for the control samples as seen in Table 3, Table 4 and Fig. 2. This implies that the first hypothesis was not supported and it confirms the findings of Burridge and Linden (2016).

Further findings of the investigation implied that:

(**i**) By increasing the mass concentration the elapsed time until 0 °C decreased. This was observed for most samples on Table 3 and Table 4 with few anomalies that might be explained by the supercooling effect. Significant differences were observed between control samples and samples with 5 g of salt. Additionally, (**ii**) both samples shown on Fig. 2 exhibited some unusual behavior around 4 °C which can be attributed to water having the highest density. Lastly, (**iii**) rate of change of temperature for $125\,\mathrm{mg\,mL^{-1}}$ samples was greater and thus resulted in a steeper temperature curve in comparison to the control.

Considering all of the previous conclusions, it should be noted that the Mpemba was not observed for any of the samples, although it was very close to being observed for the $125\,\mathrm{mg\,mL^{-1}}$ samples in Fig. 3. This, nonetheless, implies that the experiment findings do not support the second hypothesis as well, in contrast to the studies conducted by Ibekwe and Cullerne (2016) and Katz (2009), where the effect was observed.

## 4.2 Evaluation

Results of the study imply that Mpemba effect was not observed for any values of mass concentration used which is in support of findings by Burridge and Linden (2016) and Burridge and Hallstadius (2020). However, since the effect was very close to being observed for some samples, it stands to reason that possible limitations might have contributed

Considering the design of the experiment, it may be noted that the possible limitations could arise due to a couple of factors. Firstly, the experiment used tap water for all of its measurements. Due to impurities, tap water might have had a different freezing temperature and might have behaved differently as a result of that. Using deionised water instead could give more concrete results, as it would allow for studying the effect of NaCl (table salt) without the presence of other elements. Secondly, using multiple instruments to measure the same sample would ensure that error does not occur due to miscalibration. Additionally, the placement of the thermistor inside the sample might influence the temperature since different parts of the liquid have different temperatures. For the sake of repeatability, the thermistor was placed approximately in the middle of the sample 1 cm. Lastly, by using different freezing points (where the first ice particles form, 0 °C or through definition involving latent heat) different results may be obtained. In as such, using a reference point, as in Ibekwe and Cullerne (2016), might lead to the Mpemba effect being observed. This was, however, avoided to stay more in line with the original research paper.

In addition to the previously noted, future studies could explore the effects of changing volumes, having a greater range of initial temperatures and using different solutes like Mg. For instance, changing a volume from 40 mL to 80 mL would impact the cooling rate, and thus might give rise to different behaviors of temperature curves. Similarly, expanding the range of initial temperatures might help identify instances in which the effect happens. Finally, exploring the approach using other solutes might prove beneficial since it considers how other elements might impact the nature of water.

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# Appendices

## Apparatus Construction

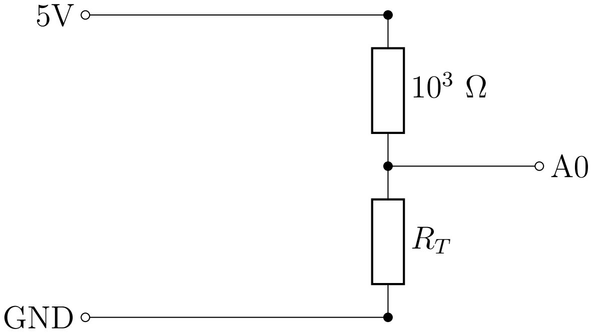

**Figure 4:** Circuit of resistor and thermistor $R_T$ connected to Arduino

| Temperature (°C)<br>(±0.1 °C) | Resistance (Ω)<br>(±1 Ω) |
|---|---|
| 22.6 | 1330 |
| 11.3 | 2140 |
| 9.5 | 2320 |
| −13.7 | 7070 |

**Table 5:** Values of $R_T$ at specific temperatures

From Fig. 4, whole circuit has a resistance $R_{tot} = R + R_T$, where $R$ is resistor with nominal resistance $1\,\mathrm{k\Omega}$ (experimentally $989\,\Omega$) and $R_T$ is the thermistor. Circuit has the potential difference $V_{arduino}$, thus:

$$V_{arduino} = I \cdot \frac{1}{R + R_T} \implies I = \frac{V_{arduino}}{R + R_T} \tag{4.1}$$

Expressing for $R_T$ and using the expression above

$$V_T = I \cdot R_T \tag{4.2}$$

$$V_T = \left(\frac{V_{arduino}}{R + R_T}\right) \cdot R_T \implies V_T = V_{arduino} \cdot \frac{R_T}{R + R_T}. \tag{4.3}$$

Using the guide provided by Recktenwald (2010), the Steinhart-Hart $\beta$ Parameter equation was used to approximate the resistance-temperature relationship for the said thermistor:

$$R(T) = R_0 \cdot e^{B(1/T - 1/T_0)} \tag{4.4}$$

where $R_0$ is the initial resistance and $T_0$ is the initial temperature from Table. 5, $R$ is theoretical resistance and $T$ is the corresponding temperature for that resistance.

$$\begin{aligned}
\frac{R}{R_0} &= e^{B(1/T-1/T_0)} \\
\ln\frac{R}{R_0} &= \frac{B}{T} - \frac{B}{T_0} \\
\ln\frac{R}{R_0} + \frac{B}{T_0} &= \frac{B}{T} \\
T &= \frac{B}{\ln\left(\frac{R}{R_0}\right) + \frac{B}{T_0}} \\
T &= \frac{T_0 \cdot B}{T_0 \cdot \ln\left(\frac{R}{R_0}\right) + B}
\end{aligned}$$

or

$$\begin{aligned}
T &= \frac{B}{\ln\left(\frac{R}{R_0}\right) + \ln\left(e^{B/T_0}\right)} \\
T &= \frac{B}{\ln\left(\frac{R}{R_0} \cdot e^{B/T_0}\right)}.
\end{aligned}$$

Finally, rearranging and lastly subtracting 273.15 to convert from Kelvins into Celsius:

$$T = \frac{B}{\ln\left(\frac{R}{R_0 \cdot e^{-B/T_0}}\right)} \implies \boxed{T = \frac{B}{\ln\left(\frac{R}{R_0 \cdot e^{-B/T_0}}\right)} - 273.15.} \tag{4.5}$$

Coefficient $B$ was obtained by using website *Thermistor Calculator* (2017) and found to be $B = 3540.94$ for values shown in Table. 2.

Using these formulas in (**??**) and (4) Python code was written (see **Appendix: Python Code**). It firstly uses Arduino apparatus to find $R_T$, thermistor resistance, and then converts $R_T$ into temperature by using the formulas outlined above.

## Arduino Code

```

// Declare the input pin at 0
int analogPin = 0;


void setup() {
  // Begin the code
  Serial.begin(9600);
}

// Main code that runs repeatdely
void loop() {
  // Read the raw data on analogPin
  float input = analogRead(analogPin);
  // Convert the raw data value (0 - 1023) to voltage (0.0V - 5.0
     V)
  double voltage = input * (5.0 / 1024.0);
  // Write the voltage value
  Serial.println(voltage, DEC);
  // Wait for 10sec, then repeat the code above.
  delay(10000);
}
```

## Python Code: Data Collection from Arduino

```
import serial
import math

# Serial port of Arduino (on Win it's COM5)
arduinoPort = 'COM3' # Connect to Arduino port
baud = 9600 # Arduino runs at 9600
fileName = 'analogData.csv' # Name of the CSV file fro data
    collection
resistor = 989 # Resistor has nominal resistance 1000ohms but
    experimentally 989ohms
voltageArduino = 5 # Voltage of Arduino
B = 3540.94 # Coefficient value obtained from Steinhart-Hart
nominalResistance = 1330 # at 22.6 Celsius

ser = serial.Serial(arduinoPort, baud)
print('Connected␣to␣Arduino␣port:' + arduinoPort)
file = open(fileName, "a")
print("Created␣file")

# Initial declaration of variables
line = 0
temperature = 0

def temperatureCalculation(input):
    temperature = B / math.log(thermistorResistance / (
        nominalResistance * math.exp( -B / 295.8))) - 273.15 #
        Steinhart-Hart B Formula
    return temperature

while 0 <= temperature:
    getData = str(ser.readline().decode('utf-8')) # Using decode
        since Python 3
    voltageThermistor = float(getData[0:][:-2]) # Curent line
        converted to float for VoltageTwo (thermistor)
    thermistorResistance = (voltageThermistor * resistor) / (
        voltageArduino - voltageThermistor)
    temperature = temperatureCalculation(thermistorResistance) #
        Pass the value of thermistorResistance into function
        defined above and return value of temperature
    print(voltageThermistor, temperature)

    file = open(fileName, "a")
```

```
    file.write(str(voltageThermistor) + ',' + str(temperature) + '
       \n') # Write thermistor voltage into CSV file
    line = line + 1

print('Data␣collection␣complete!')
file.close()
```